Karlsruher Institut für Technologie

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Sommersemester 2026

Seminar on algorithmic properties of groups

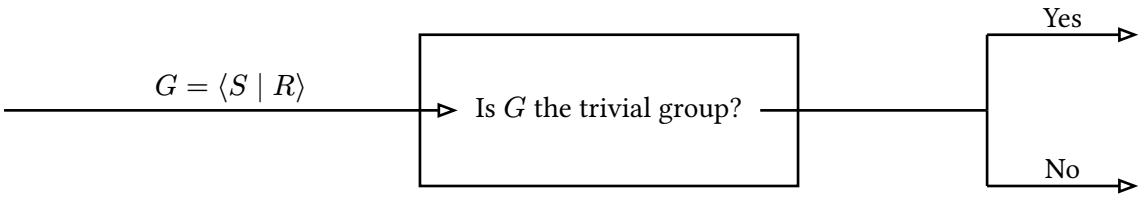
Premeeting: Wednesday, 18th February 2026 at 14:00
Building 20.30, Seminar Room −1.015 (Underground floor)

Seminar dates: On Fridays at 9.45. Depending on preferences, the second half can be organized as a Blockseminar.

Language: English

Prerequisites: Basic knowledge about groups will be assumed.

Registration: To register or if you have any question, please send an email to marco.amelio@kit.edu and matteo.migliorini@kit.edu.



Contents

In group theory, there are many natural *decision problems* that one can consider, such as:

- **Isomorphism problem**: Given two group presentations $G = \langle S \mid R \rangle$ and $H = \langle S' \mid R' \rangle$, are G and H isomorphic?
- **Word problem**: Given $G = \langle S \mid R \rangle$ and a word w in the generating set S, can we determine if it represents the trivial element?
- **Membership problem**: Given a word w and a subgroup $H < G$, can we determine if w represents an element of H?

Perhaps surprisingly, it turns out that the majority of decision problems for groups are *undecidable*, meaning that no algorithm is guaranteed to determine an answer in finite time. In fact, the isomorphism problem is not even decidable in the case where H is trivial, meaning that there is no algorithm that can recognise presentations of the trivial group.

In this seminar, we will introduce the notion of Turing machine, that can be used to give a formal definition of decidability. We will then introduce some classic decidability problem for groups, focusing in particular on the word problem.

We will then see that decision problems are central in active areas of research in group theory. One such example is the Boone–Higman conjecture, which aims to characterize groups that have solvable word problem as the ones that embed in a finitely presented simple group.

References

The seminar plan will mostly follow the seminar course “Decision problems in groups” organized by Clara Löh, Matthias Uschold and Franziska Hofmann during the Winter Semester 2023/2024 in Regensburg.

The main reference is the book by Rotman “An Introduction to the Theory of Groups”; additional literature may be provided for specific talks.